Физика горения и взрыва, 1996, т. 32, № 5

ITERATIVE SOLUTION OF THE RADIATION TRANSPORT EQUATIONS GOVERNING SPREAD OF FIRE IN WILDLAND FUEL

F. A. Albini

Mechanical Engineering Department Montana State University, Bozeman MT 59717

INTRODUCTION

To model mathematically the spread of fire in wildland fuels, one must predict the rate at which heat is transferred from the fuel that is burning to the fuel that has yet to be ignited, for this is the process that governs the rate of fire spread [1, 2]. Wind-aided fire spread, and spread upslope in uneven terrain, give rise to the situation in which the hot flame gases from the burning zone can impinge directly upon the unignited fuel. In these cases heat transfer to the unignited fuel must involve convective and radiative mechanisms. Models for such phenomenology have been proposed, ranging from conceptual skeletons [3] to encyclopedically encompassing fluid mechanical simulations [4]. But, as many have realized, the dominant mechanism of heat transfer is often radiation [5–11] for the simple reason that the movement of air is toward the burning zone rather than away from it, at least in the near vicinity of the fire edge where rapid heating of unignited fuel is taking place. This situation obtains even for wind-aided fire spread whenever the flame structure stands erect from the fuel surface rather than being blown through it.

The most recent model which attempts a comprehensive treatment of radiation-dominated spread of fire was proposed by the author [12, 13] and tested on laboratory fires in moist excelsior (thin filamentous wood shavings) under calm conditions. It treated a single-component uniform stratum of fuel resting on an adiabatic surface that was a perfect black radiator at ambient temperature. The model predicts rate of fire spread and shape of the ignition interface given the effective radiometric temperature of the burning zone and the free flame standing above the fuel bed and the height and tilt angle of the free flame, along with a sufficient description of the properties of the fuel bed. The ignition interface and the flame are treated as uniform diffuse sources.

The model was modified to allow for the existence of a zone beneath the fuel layer that does not participate in the spread governing process, but contains a planar flame front attached to the ignition interface in the uniform layer, and attenuates radiation with an extinction coefficient different from that of the upper fuel layer. The model so extended was applied to experimental field data from crown fires in immature Jack pine, with very encouraging results [14]. The model predicted quite accurately the spread rates of all the experimental fires, using flame heights and tilt angles scaled from a single fire by a simplified flame structure model [15] and constant radiometric temperatures for burning zones and free flames. Modification and extension of this latest version of the model is the objective of the present research. Objectives of the effort include making the model permissive of more than one fuel stratum, inclusion of a method of homogenizing a fuel layer consisting of many fuel component sizes and characteristics

into an equivalent unform single size class, and closure of the model by implementing prediction of flame height and tilt angle consistent with spread rate, wind, and fuel data.

MODIFICATION AND EXTENSION OF MODEL

The latest version of the model, like all the other versions before it, addresses the rate of spread of a line fire as a problem with bilateral symmetry. This feature will not be changed in the present effort. The model at present deals with only one fuel component in a single homogeneous stratum representing, for example, the live crown layer. An underlying layer, if present, is treated as thermally inert, serving only as an attenuating medium for the propagation of radiation and containing a vertical flame front that radiates with the power density of the burning zone of the fuel layer. It is proposed to relax this idealization and to simulate the ignition interface from the "ground" surface to the top of the upper fuel layer. Inclusion of three uniform layers is contemplated for this effort: a surface cover layer, a subcanopy layer, and a canopy layer. The amount of energy required to raise a unit volume of fuel bed to ignition temperature will differ for the three layers, as will their radiation extinction lengths, so the radiation transport solution algorithm and ignition interface location iteration must be generalized. Finally, the model now requires that the height and tilt angle of the free flame above the fuel bed be specified, rather than predicting these properties from the wind speed, fire intensity, and fuel layer height. An approach to accomplishing these extensions is outlined in this section, with details relegated to appendices.

Fuel Layer Homogenization. The different amounts, sizes, moisture contents, etc., of the fuel components making up each of the layers must be combined into an equivalent single-component surrogate fuel distribution. To this end an algorithm has been formulated that adds weighted contributions of each of the different fuel components within a layer and derives a surrogate fuel layer description that contains only one fuel component. Some quantities that the formulation of this transcription nominally keeps constant are:

1) Total fuel surface area per unit volume of fuel bed (optical extinction parameter),

2) Fraction of fuel bed volume filled with fuel,

3) Heat released (more generally, mass average values of intrinsic properties) due to fuel consumption in the burning zone creating the free flame at the leading edge of the fire.

In order to compute the heat released it is necessary to specify the fraction of each fuel component loading that will be burned at the leading edge of the fire. For this purpose the following idealized model is proposed: Each fuel element will experience a reduction of its effective diameter that is inversely proportional to the heat energy required to ignite a unit volume of its mass. This hypothesis leads to an expression [16] for the fractional reduction in diameter of a fuel element with dry mass density ρ, surface area/volume ratio σ, moisture content M of the form:

$$\Delta D/D = \min(a\sigma/\rho(1+bM), 1) \tag{1}$$

where b is expected to be approximately 5 and the value of a must be about 3/8 for SI units of measure. This yields a prediction for the fractional mass loading of size class j burned at the leading edge of a fire front, g_j,

$$g_j = 1 - (1 - (\Delta D/D)_j)^2 \tag{2}$$

where $(\Delta D/D)_j$ is computed from (1) using σ, ρ, and M for component j.

The factor g_j represents the fraction of the loading of fuel component j that participates in propagating the fire. Applying this interpretation of g_j and imposing the three conditions listed above gives the recipe for reducing a multicomponent fuel collection to an equivalent, single component surrogate fuelbed. Note that this consolidation does not distinguish between live and dead fuel components; only the size, density, and moisture content is used to establish each fuel component's contribution to the fire spread process. Appendix A provides the remaining details of this algorithm.

Generalized Radiation Transport Solution. Determination of the rate of spread of the fire and, simultaneously, the shape of the ignition interface surface is the core of the model. To derive the ignition interface shape requires specification of the radiative power input to the fuel bed from the free flame above it and from the burning zone within it. As the ignition interface separates the burning zone from the unignited fuel, it is also an isotherm of the solid fuel temperature field, at the ignition temperature. Clearly an iterative solution is indicated. To start the iteration, a trial fire spread rate, along with a free flame geometry and an interface shape are posited. Using these data, the radiation transport equation is solved to discover the fuel temperature distribution. The ignition interface shape and spread rate are then adjusted and the process repeated until convergence is achieved.

The details of the iteration scheme for a single uniform layer have been described by the author [12, 13] and will not be repeated here. In brief, a coordinate system is fixed to the moving ignition interface, the radiation incident from the interface and the free flame are distributed through the fuel bed using an extinction coefficient based on the probabilisty of physical blockage of the radiation pathway by solid fuel particles. Fuel particles are considered to be uniformly distributed with random orientation, and to be thermally thin and radiometrically black. This gives rise to an energy density deposition rate that declines exponentially with distance from the source through the fuel bed. This energy density is translated into a fuel temperature, based on the fuel's moisture content, mass density, specific heat capacity, and the fraction of fuel.bed volume filled with solids. This temperature distribution represents a distributed source of radiant heat, which tends to smooth the absorbed energy density distribution. Convective cooling of the fuel particles modifies the redistribution process but does not change it fundamentally. By formulating the solution in terms of integrals over the fuel bed volume and working in terms of differences between successive iterations, efficient algorithms were developed and solutions were found to converge promptly. Generalization of this iteration and the adjustment of spread rate and interface shape to a fuel bed with properties that vary with vertical location, is described in more detail in appendix B.

Flame Geometry Prediction and Model Closure. To close the model tested by Albini and Stocks [14] the flame height and tilt angle must be related to the fire spread rate. To this end, the heat release per unit area is multiplied by the predicted spread rate to yield the line intensity of the fire. This intensity is used in a simplified version of a model for the wind-blown flame from a line fire [15] to predict flame height and tilt angle. The model predicts a relationship between fire line intensity, I_f, flame height, H_f, and average windspeed incident on the flame structure, U, of the form

$$H_f = AI_f/U \tag{3}$$

where A is an empirical constant estimated to be approximately 0.005 m^3/kJ. The average windspeed acting on the flame is that which would exist over the flame height range if there were no flame present. To calculate this, one requires only the value of the windspeed at the

top of the canopy layer, U_c, and the height of the forest stand, H_c, according to the model [17]. The formula for this average windspeed given by Albini and Stocks [14] can be simplified to the form:

$$U/U_c = (1 + 0.36H_c/H_f)\ln(1 + H_f/(0.36H_c)). \tag{4}$$

Using (3) to express U/U_c as (AI/H_cU_c) makes (H_f/H_c) a transcendental function of this dimensionless group. The angle, α, describing the flame tilt to leeward is established by the Froude number based on flame height:

$$\tan^2\alpha = 3U^2/2gH_f \tag{5}$$

which allows the imposition of a minimum tilt angle to avoid singular behavior of flame height at low windspeed.

The rate of spread is parametrically a function of the flame height, H_f, and tilt angle, α, and hence of the intensity, I_f. A graph of the spread rate predicted versus the intensity upon which it is based would be a monotone increasing curve with negative curvature and a nonzero spread rate at zero intensity. This apparent contradiction arises because the radiation from the burning zone is modeled as independent of the intensity of the fire. But it assures that there will be an intersection of the graph of spread rate versus fire intensity and the locus of intensity versus spread rate established by the burning of all the fuel within the single-component equivalent fuel layers as the fire burns over it. The negative curvature of the spread rate curve also insures that the intersection represents a solution that is stable to iteration. That is, the intensity given by a spread rate different from that at the intersection of the two curves will give rise to a spread rate that is closer to the intersection point. This means that a straightforward sequential iteration should eventually yield a stable and meaningful solution.

Finally, the semi-empirical parameters T_B and F that describe the radiation intensity from the burning zone and the free flame can probably be assigned the constant values

$$T_B = 1050\,\mathrm{K}, \qquad F = 0.40 \tag{6}$$

based on the findings of [13, 14]. At least these values can be used as initial estimators in comparing predictions to field data, with at most small shifts to be expected.

SUMMARY

A model for the rate of spread of a wildland fire in the radiation-dominated condition has been designed. The model is a generalization and extension of one previously tested which was restricted to a single participating uniform crown layer with a nonparticipating subcanopy stem space. Generalization extends the model domain to include three uniform participating layers, and further generalization could make the vertical distribution of fuel bed properties arbitrary. A set of algorithms has been designed to implement the generalized model, and they have been implemented as computer code. The components of the model remain to be assembled into a single working computer program and calibrated/tested against field data. If the model performs satisfactorily in these and other tests to be devised, it will be embodied in a package of *PC*-based software that would facilitate its application in a variety of situations with a minimal demand for the detailed data that it requires as a research tool.

ACKNOWLEDGEMENT

The author gratefully acknowledges not only financial support of this endeavor by the Canadian Forest Service through the "Green Plan", but technical contributions and encouragement from Canadian Forest Service Fire Research professionals. Special thanks in this regard are due to Brian J. Stocks, Bruce D. Lawson, and Martin E. Alexander.

APPENDIX A: EQUIVALENT SINGLE SIZE CLASS FUEL BED

Purpose. This appendix describes an algorithm that combines loadings of different size classes of fuel, with different intrinsic and extrinsic properties, into a single size class fuel bed description. In generating the single size class equivalent fuel bed, the intention is to preserve many gross features of the original fuel bed — especially those that influence or control the fire spread rate.

The single size class fuel bed is intended to contain the quantity of fuel that is burned in the leading edge of a spreading fire, producing the same fire edge intensity as the multicomponent fuel bed. While this cannot be mathematically assured, it is assumed that the expression for fractional diameter reduction posited below describes this process at least to an acceptable level of approximation. Then the other quantities held constant are consistently transcribed from a multicomponent fuel bed to its single size class equivalent.

The quantities that are maintained invariant in this transcription are:

1) Total fuel surface area per unit volume of fuel bed.
2) Fraction of total fuel bed volume filled with fuel.
3) Mass average values of intrinsic properties of fuels consumed at the fire leading edge.

Nomenclature. Let subscript i designate a fuel component and subscript e the equivalent single component fuel bed quantity. The glossary of fuel and fuel bed descriptors is given below; SI units are used throughout.

W	Ovendry fuel loading, kg/m^2
M	Fuel moisture content, fraction ovendry mass
D	Fuel element diameter (elements assumed to be right circular cylinders), m
σ	Surface area/volume ratio, m^{-1}
ρ	Ovendry mass density, kg/m^3
h_c	Specific heat of combustion, ovendry ash-free basis, J/kg
m	Mineral ash content, fraction ovendry mass
δ	Fractional diameter reduction of fuel element in burning zone at the leading edge of a spreading fire
C	Specific heat capacity, J/(kg · K)

Equivalent Fuel Bed Algorithm. The transcription begins with calculation of the fractional diameter reduction of each fuel component in the burning zone at the leading edge of a spreading fire.

$$\delta_i = \min\left(a\sigma_i/(\rho_i(1 + bM_i)), 1\right). \tag{A1}$$

The quantities a and b in this expression are constants to be determined empirically. From thermodynamic considerations, the parameter b should have a numerical value of about 5.0, and this is the value used. For SI units, a value of 3/8 for parameter a corresponds to complete consumption of 0.5 mm diameter parts with 100 % moisture content, 1.0 mm diameter parts

with 40 % moisture content, 2.0 mm diameter parts with 10 % moisture content, and 3.0 mm diameter bone dry parts, if each has an ovendry density of 500 kg/m^3.

Using this quantity, the fraction of the loading of each distinct fuel element enumerated in the multicomponent fuel bed that is consumed in the leading edge of the fire, q_i, is to be found from

$$g_i = 1 - (1 - \delta_i)^2 \tag{A2}$$

which allows calculation of the ovendry loading of the equivalent fuel bed's single component:

$$W_e = \sum g_i W_i. \tag{A3}$$

It is understood that this is the loading of fuel that is promptly burned in the leading edge of the fire in either fuel bed, while in the multicomponent fuel bed the fraction of the loading of each element i that burns is given by q_i. Thus the equivalent loading would have the moisture content M_e, where

$$M_e = \sum g_i W_i M_i / W_e. \tag{A4}$$

To preserve the fractional volume of the fuel bed that is filled with solid fuel, the equivalent particle mass density must be adjusted to:

$$\rho_e = W_e / \sum (W_i / \rho_i). \tag{A5}$$

With the equivalent mass and volume loadings defined, we preserve the fuel surface area per unit volume of fuel bed by selecting the equivalent particle surface/volume ratio according to:

$$\sigma_e = (\rho_e / W_e) \sum (W_i / \rho_i) \sigma_i. \tag{A6}$$

Intrinsic fuel particle properties used in the computation of heat release and in determining the energy required for ignition are then defined in terms of the contributions of the multicomponent fuel bed elements.

$$m_e = \sum g_i W_i m_i / W_e, \tag{A7}$$

$$h_{ce} = \sum g_i W_i (1 - m_i) h_{ci} / (W_e (1 - m_e)), \tag{A8}$$

$$M_e = \sum g_i W_i M_i / W_e, \tag{A9}$$

$$C_e = \sum g_i W_i C_i / W_e. \tag{A10}$$

APPENDIX B: GENERALIZED RADIATION TRANSPORT SOLUTION

Coordinates and Nomenclature. The radiation transport problem to be solved may be posed in the following terms: In a Cartesian coordinate system fixed to the steadily advancing linear flame front, let x' measure the longitudinal distance from the fire edge, measured perpendicular to it, positive in the direction of fire spread. Let z' be the upward distance measured locally perpendicular to the terrain surface, and y' the lateral position. For the moment, we consider only flat terrain, so z' is measured vertically. We assume that a quasisteady state has been achieved, so that in the coordinate system described, the temperature distribution is stationary in time. We seek a solution to the equations of heat transfer in the porous fuel bed to discover the rate of spread of fire that satisfies simultaneously two criteria:

1) The interface separating burning and unignited fuel is an isotherm at the ignition temperature for the solid fuel particles and

2) The rate of spread yields a fire intensity that results in the flame structure causing that rate of spread. The effect of wind is exerted through its influence on flame height and the flame tilt angle that it induces. The following nomenclature is employed.

B	Stefan–Boltzmann constant = $5.67 \cdot 10^{-8}$ W/(m$^2 \cdot$ K^4)
$b(z)$	Cold boundary asymptote parameter, equation (B20)
$C(z)$	Optical opacity parameter (dimensionless) = $\sigma(z)\beta(z)\delta/4$
$C_e(z_1, z_2)$	Mean (or effective) value of $C(z)$ over the height range z_1 to z_2
F	Intensity of flame excess radiation relative to that from burning zone
H	Flame height normalized by fuel bed height (δ)
h	Newtonian film heat transfer coefficient for fuel particles, W/(m$^2 \cdot$ K)
I	Intensity of radiation field, W/(m$^2 \cdot$ sterad)
I_B	Intensity of radiation field at burning zone interface, W/(m$^2 \cdot$ sterad) = BT_B^4/π
i	Normalized radiation intensity above background level = $I/I_B - (T_a/T_B)^4$
J_F	Dimensionless density of rate of radiant heat absorption from free flame
J_I	Dimensionless density of rate of radiant heat absorption from burning zone
J_V	Dimensionless density of rate of radiant heat absorption from fuel bed
(l', m', n')	Direction cosines of a pencil of radiation
l_I	Distance from a point in the fuel bed to a point on the ignition interface
l_V	Distance from one point in the fuel bed to another
$q(x, z)$	Energy per unit volume of fuel bed absorbed by the solid fuel within it, J/m^3
$q_b(z)$	Energy per unit volume of fuel bed required to raise solid fuel within it to T_b, J/m^3
$q_d(z)$	Energy per unit volume of fuel bed required to desiccate solid fuel within it, J/m^3
$q_i(z)$	Energy per unit volume of fuel bed required to ignite solid fuel within it, J/m^3
$Q(x, z)$	Normalized distribution of energy absorbed per unit volume, $q(x, z)/q_i(z)$
Q_b	$q_b(z)/q_i(z) \neq$ fn(z)
Q_d	$q_d(z)/q_i(z) \neq$ fn(z)
R	Quasisteady rate of fire spread, m/s
s	Distance along the ray path of a pencil of radiation
T	Temperature of (thermally thin) solid fuel particles
T_a	Ambient temperature, K
T_b	Boiling temperature of water, K
T_B	Effective radiometric temperature of burning zone, K
T_i	Ignition temperature of solid fuel, K
(x', y', z')	Dimensional Cartesian coordinates
(x, y, z)	Dimensionless Cartesian coordinates (x'/δ, y'/δ, z'/δ)
$x_I(z)$	Dimensionless coordinate of ignition interface at height z
α	Flame front tilt angle from vertical (positive in direction of fire spread)
$\alpha_2(z)$	Interface tilt angle from vertical = $dx_I(z)/dz$
$\beta(z)$	Fraction of fuel bed volume filled with solid particles at height z
δ	Total height of fuel bed, m
γ	Parameter in equation (B2) = $4h(T_i - T_a)/BT_B^4$
Θ	Normalized radiometric temperature variable = $(T/T_B)^4 - (T_a/T_B)^4$
$\Lambda(z)$	Dimensionless ratio $Rq_i(z)/BT_B^4$
$\sigma(z)$	Fuel particle surface area/volume ratio, m^{-1} at height z
τ	Dimensionless temperature rise = $(T - T_a)/(T_i - T_a)$

Formulation. Following the earlier model development [12, 13], the equation describing the radiation intensity field can be difference from the background level, normalized, written in the form (see definition of i above)

$$(l'\partial/\partial x + m'\partial/\partial y + n'\partial/\partial z)i = C(z)(\Theta - i). \tag{B1}$$

The density of energy absorbed evolves according to

$$(\Lambda(z)/C(z))\partial Q/\partial x = -(1/\pi)\iint_{4\pi} id\omega' + 4\Theta + \gamma\tau \tag{B2}$$

and the temperature is fixed by the heat absorbed, so Θ and τ depend upon Q. The integral of i over 4π steradians gives rise to contributions from the free flame, from the burning zone, and from the heated particles within the fuel bed. This can be seen explicitly by examining the solution of equation (B1).

The radiation field, i, can be expressed in terms of the distributed volumetric radiation source term, Θ, and any boundary sources:

$$i(s) = i_0 \exp(-r(0,s)) + \int_0^s \Theta(s')C(s')\exp(-r(s',s))ds'. \tag{B3}$$

Here

$$r(s',s) = \int_{s'}^{s} C(s'')ds'' \tag{B4}$$

is the extinction exponent between locations s' and s which lie along the direction fixed by the pencil of radiation being considered. Location 0 is on the boundary of the fuel bed and i_0 represents the radiation incident upon the boundary at the point where the pencil of radiation enters the fuel bed along the direction considered. Thus the integral in equation (B2) can be written as

$$\iint_{4\pi} id\omega' = J_F + J_I + J_V \tag{B5}$$

where the three contributions arise from heating by radiation from the flame (J_F), from the ignition interface (J_I), and from the heated particles distributed throughout the volume of the fuel bed (J_V).

The compact integral expressions for J_F and J_I presented in [12] require only slight modification for validity when the extinction coefficient varies with z. The fact that the opacity parameter, C, depends only upon the height makes it possible to account for its variation by means of an "effective" value that can be tabulated in a manner consistent with the discretization of the fuel bed. The generalized expression for J_F becomes:

$$J_F = 2F[1 - (T_a/T_B)^4]\int_{\varphi_1}^{\varphi_2} f_1((1-z)C_e(1,z)/\cos\varphi)d\varphi. \tag{B6}$$

Here the integral limits are the angles (measured counterclockwise from vertical) to the tip and the base of the flame, respectively, as viewed from the point within the fuel bed where J_F is evaluated.

$$\varphi_1 = \tan^{-1}((x - x_I(1) - H\tan\alpha)/(1 + H - z)), \tag{B7}$$

$$\varphi_2 = \tan^{-1}((x - x_I(1))/(1 - z)). \tag{B8}$$

The generalized expression for J_I is also quite similar to the original form:

$$J_I = 2[1-(T_a/T_B)^4]\int_0^1 f_1(C_e(z,z'')l_I)\{x - x_I(z'') - (z-z'')\tan\alpha_2\}dz''/l_I^2 \tag{B9}$$

where

$$l_I^2 = (x - x_I(z''))^2 + (z-z'')^2 \tag{B10}$$

and α_2 is evaluated at height z'' for the integration.

In (B6) and (B9) the tabulated function f_1 arises from the lateral integration. Actually two functions of the form

$$f_N(\xi) = \int_0^{\pi/2} \exp(-\xi/\cos\varphi)d\varphi/\cos^N\varphi; \quad N = 0,\ 1 \tag{B11}$$

are tabulated for lookup during execution of the algorithm, as the $N = 0$ case occurs in the expression for J_V:

$$J_V(x,z) = 2\int_0^1 C(z'')\int_{x_I(z'')}^{\infty}\Theta(x'',z'')f_0(C_e(z,z'')l_V)dx''dz''/l_V. \tag{B12}$$

Here

$$l_V^2 = (x-x'')^2 + (z-z'')^2. \tag{B13}$$

Integration of (B2) provides the distribution of heat absorbed by the solid fuel

$$Q(x,z) = \{C(z)/\Lambda(z)\}\int_x^{\infty}[(J_I + J_F + J_V)/\pi - 4\Theta - \gamma\tau]dx'' \tag{B14}$$

which in turn provides the distribution of solid particle temperatures:

$$Q \leqslant Q_b: \qquad T = T_a + (T_b - T_a)Q/Q_b, \tag{B15}$$

$$Q_b \leqslant Q \leqslant Q_d: \qquad T = T_b, \tag{B16}$$

$$Q_d \leqslant Q: \qquad T = T_b + (T_1 - T_b)(Q - Q_d)/(1 - Q_d) \tag{B17}$$

from which the functions τ and Θ are readily constructed. Once the distribution Q is determined, the temperature field is established, and hence the radiation intensity field, $i(x,z)$. The iteration procedure proposed for the solution of this problem is described in the next section.

Inner Iteration. Making use of the fact that the distributions J_I and J_F can be evaluated explicitly once the interface shape and the flame geometry are specified, an iteration can be formulated based upon an initial estimator for the distribution Q, to which the outcome of the iteration is not sensitive. Of course, the more accurate is the initial estimator for Q (denoted $Q^{(1)}$), the more promptly will the iteration converge. Formally, the iteration can be specified as:

$$Q^{(\nu)} = Q^{(1)} + \sum_{n=1}^{n=\nu}[Q^{(n+1)} - Q^{(n)}] \tag{B18}$$

where the initial distribution $Q^{(1)}$ is found from the sum of a cold boundary limit value plus a

direct integration of the distributions J_I and J_F:

$$Q(x_{cold}, z) = (C(z)/b\Lambda(z)\pi) \int_0^1 (J_F + J_I)\Big|_{(x_{cold} - b^{-1}\ln(u), z)} du \tag{B19}$$

where b, the parameter fixing the asymptotic behavior of Q, is given by [13]

$$b = b(z) = [16C(z)T_a^3(T_b - T_a)/Q_b\Lambda(z)T_B^4][1 + h/4BT_a^3]. \tag{B20}$$

Using the cold boundary values, the contributions of J_I and J_F to the Q distribution can be captured in the initial estimator for $Q(x, z)$ by:

$$Q^{(1)}(x, z) = Q(x_{cold}, z) + (C(z)/\pi\Lambda(z)) \int_x^{x_{cold}} (J_I + J_F)_{(x'', z)} dx''. \tag{B21}$$

If the difference between any of the distributions from iteration (n) to iteration $(n + 1)$ is denoted by Δ_n so that (using Q for example)

$$\Delta_n Q = Q^{(n+1)} - Q^{(n)} \tag{B22}$$

we have for the evolution of the Q field iterands the form

$$\partial\Delta_n Q/\partial x = -(C(z)/\Lambda(z))\Delta_n(J_V/\pi - 4\Theta - \gamma\tau) \tag{B23}$$

in which the difference operator commutes with J_V so the factor Θ in (B12) becomes the difference $\Delta_n\Theta$. This iteration sequence is closed by specifying that the temperature field (and hence the Θ and τ fields) for iteration $(n + 1)$ is to be found by using the field $Q^{(n)}$ in (B14)–(B16).

Outer Iteration. With an interface shape, $x_I(z)$, and a flame geometry, (H, α), specified, the inner iteration described above can be prosecuted to establish a distribution of heat absorbed, $Q(x, z)$. This distribution should satisfy the condition

$$Q(x_I(z), z) = 1{,}0. \tag{B24}$$

which would assure that the ignition interface described an isotherm at the ignition temperature. If this relationship is not satisfied, it will be necessary to adjust the interface shape and/or the rate of fire spread to achieve this condition. The flame height and tilt angle are also related to the spread rate and ambient wind field, as described in the text, so if the flame spread rate, R, is changed, then the fire line intensity is changed (see appendix A)

$$I_f = RW_e(1 - m_e)h_{ce} \tag{B25}$$

and the flame geometry must be adjusted according to the relationships given in equations (3)–(5).

Generalizing [12], the following adjustments are prescribed for the interface shape and spread rate. Denote by $x_I^{(k)}(z)$ the k-th ignition interface shape used, and determine by interpolation and extrapolation the shape of the contour $x_*^{(k)}(z)$ upon which the heat absorbed is equal to the average value over the interface $x_I^{(k)}(z)$. That is, implicitly, $x_*^{(k)}(z)$ is defined by

$$Q^{(k)}(x_*^{(k)}(z), z) = \overline{Q(x_I^{(k)}(z), z)} \tag{B26}$$

where the overbar indicates a spatial average. With $x_*^{(k)}(z)$ so defined, the interface shape can

be revised according to:

$$x_I^{(k+1)}(z) = (x_I^{(k)}(z) + x_*^{(k)}(z))/2. \tag{B27}$$

The rate of spread can be revised according to:

$$R^{(k+1)} = (1 + \overline{Q^{(k)}}(z))R^{(k)}/2 \tag{B28}$$

using which, the fire line intensity and hence the flame geometry can be revised.

Note that it should usually be necessary to revise the spread rate with each outer iteration. If the spread rate, flame height, and flame tilt are held constant, then $Q(x_I(z), z)$ should converge to its spatial average value if the interface shape is adjusted according to (B27). But it is a virtual certitude that this spatial average value would not be unity. To achieve this convergence it is necessary to adjust the rate of spread, for which (B28) should once again prove adequate.

REFERENCES

1. **Chandler C. et al.** Fire in Forestry. Vol. 1: Forest Fire Behavior and Effects / C. Chandler, P. Cheney, P. Thomas, L. Trabaud, D. Williams. New York: Wiley-Interscience, 1983.
2. **Albini F. A.** Wildland fires // Amer. Scientist. 1984. V. 72. P. 590–597.
3. **Albini F. A.** A physical mechanism for fire spread in brush // 11th Symp. (Int.) on Combust. Pittsburgh, PA: Combust. Inst., 1967. P. 553–560.
4. **Grishin A. M.** Mathematical Modeling of Forest Fires and New Methods of Fighting Them. Novosibirsk: Nauka, 1992.
5. **Emmons H. W.** Fire in the forest // Fire Res. Abst. and Rev. 1964. V. 5, N 3. P. 163.
6. **Hottel H. C., Williams G. C., Steward F. R.** The modeling of fire spread through a fuel bed // 3th Symp. (Int.) on Combust. Pittsburgh, PA: Combust. Inst., 1965. P. 997–1007.
7. **Thomas P. H.** Some aspects of the growth and spread of fire in the open // Forestry. 1967. V. 40, № 2. P. 139.
8. **Hottel H. C., Williams G. C., Kwentus G. K.** Fuel preheating in free-burning fires // 13th Symp. (Int.) on Combust. Pittsburgh. PA: Combust. Inst., 1971. P. 963–970.
9. **Frandsen W. H.** Fire spread through porous fuels from the conservation of energy // Combust. Flame. 1971. V. 16. P. 9.
10. **Telisin H. P. (var. Telitsin G. P.)** Flame radiation as a mechanism of fire spread in forest // Heat Transfer in Flames / N. H. Afgan, J. M. Beer (Eds). New York: John Wiley and Sons, 1974. P. 441–449.
11. **Kurbatskiy N. P., Telitsin G. P.** Theoretical and experimental analysis of the radiation mechanisms for the spread of forest fire // Characteristics of the Process of Combustion in Forests / E. V. Konev (Ed.). Krasnoyarsk, RU: USSR Academy of Sciences, Siberian Branch, 1977. P. 1–33. (Transl. by Int. Translation Center, Inc., Washington, DC).
12. **Albini F. A.** A model for fire spread in wildland fuels by radiation // Combust. Sci. Technol. 1985. V. 42. P. 229–258.
13. **Albini F. A.** Wildland fire spread by radiation — a model including fuel cooling by natural convection // Combust. Sci. Technol. 1986. V. 45. P. 101–113.
14. **Albini F. A., Stocks B. J.** Predicted and observed rates of spread of crown fires in immature jack pine // Combust. Sci. Technol. 1986. V. 48. P. 65–76.

15. **Albini F. A.** A model for the wind-blown flame from a line fire // Combust. Flame. 1981. V. 43. P. 155–174.
16. **Albini F. A., Reinhardt E. D.** Modeling ignition and burning rate of large woody natural fuels // Int. J. Wildland Fire. 1995. V. 5, N 2. P. 81–91.
17. **Baughman R. G., Albini F. A.** Estimating midflame windspeeds // Proc. Sixth Conf. on Fire and Forest Meteorology / R. E. Martin et al. (Eds). Washington, DC: Soc. Amer. Foresters, 1980. P. 88–92.

Поступила в редакцию 25/III 1996 г.